Engineering Notes

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Coordinated Turn Relations: A Graphical Representation

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I. Introduction

THE analysis of the coordinated turn is well known.[1,2] In the present Note, the relationships among the various flight parameters involved in the turn are presented in graphical forms that facilitate the delineation of "performance windows." These can, for example, be used to estimate the flight parameters that will allow the airplane to achieve a specified maneuverability, as measured by turning rate or radius of turn, without exceeding specified load factor or lift coefficient limitations.

The equations upon which the graphs are based are presented in Sec. II. The graphs themselves are given in Sec. III, along with illustrative examples of their use.

The equations, and therefore the graphs, are approximate in that they do not consider possible performance limitations due to the available power or thrust. Those limitations, which can result in a contraction of the performance windows, can be evaluated separately in any specific case if the drag coefficient is known as a function of lift coefficient and Mach number. (Power-limited turning flight is analyzed in considerable detail in Ref. 3 for the case of a parabolic drag polar.) The graphs are also approximate (although in a sense conservative) in their neglect of any lift component of the thrust. This component of the thrust, which can be significant for high-thrust aircraft operating at high angles of attack (large lift coefficients) is considered in the turn analysis of Ref. 2. It is not readily included within the framework of a graphical presentation.

II. Equations

Neglecting any lift component of the thrust, the equations governing the coordinated turn are known to be

$$L\sin\phi = \frac{W}{g}\frac{V^2}{R} \tag{1}$$

$$L\cos\phi = W \tag{2}$$

$$n = L/W = \sec\phi \tag{3}$$

$$\omega = V/R \tag{4}$$

$$L = C_L \rho V^2 S/2 \tag{5}$$

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where L is the lift, W the weight, V the airspeed, R the radius of turn, ϕ the angle of bank, n the load factor, ω the turn rate in radians per unit time, C_L the lift coefficient, ρ the air density, S the wing area, and g the acceleration of gravity. From these basic relations, the following subsidiary relations, involving dimensionless groupings of the variables, can be obtained:

$$\frac{Rg\rho}{W/S} = \frac{1}{\sqrt{n^2-1}}\frac{V^2\rho}{W/S} \tag{6}$$

$$\left(\frac{W/S}{Rg\rho}\right)^2 + \left(\frac{W/S}{V^2\rho}\right)^2 = \frac{1}{4}C_L{}^2 \tag{7}$$

$$\frac{1}{\omega}\sqrt{\frac{\rho g^2}{W/S}} = \frac{1}{\sqrt{n^2-1}}\left(V\sqrt{\frac{\rho}{W/S}}\right) \tag{8}$$

$$\frac{1}{\omega}\sqrt{\frac{\rho g^2}{W/S}} = \left[\frac{C_L^2}{4}\left(\frac{V^2\rho}{W/S}\right) - \frac{W/S}{V^2\rho}\right]^{-1/2} \tag{9}$$

III. Graphs

Equation (6) is represented by the straight lines in Fig. 1 and Eq. (7) by the curved lines. Similarly, Eqs. (8) and (9) are represented by the straight and curved lines, respectively, in Fig. 2. Each figure involves four performance parameters, of which any two determine the other two.

In order to illustrate the use of Fig. 1, consider an airplane with a wing loading of $W/S = 2000$ N/m^2 (42 lb/ft^2), operating at an altitude where $\rho = 1$ kg/m^3 (0.00194 slugs/ft^3), and determine the range of velocities for which it is possible to execute coordinated turns with R not exceeding 500 m (1640

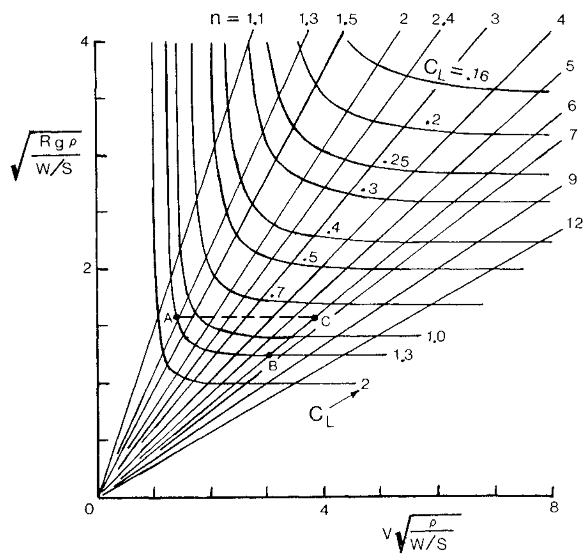

Fig. 1 R-V-n-C_L relationships.

ft), C_L not exceeding 1.3, and n not exceeding 6. The latter two limits dictate that the allowable region of Fig. 1 lies to the right of the $C_L = 1.3$ curve and to the left of the $n = 6$ line. The limitation on R restricts the allowable region further to the area below the horizontal line defined by

$$\sqrt{\frac{Rg\rho}{W/S}} = \sqrt{\frac{(500)(9.81)(1)}{2000}} = 1.57$$

Thus is established the quasi-triangular performance window ABC in Fig. 1. In particular, points A and C, through their abscissa values of 1.40 and 3.82, define the minimum and maximum speeds ($V_{\min}$ and $V_{\max}$) for which the specified performance is possible as follows:

$$V_{\min}\sqrt{\frac{\rho}{W/S}} = 1.40 \qquad \therefore V_{\min} = 1.40\sqrt{\frac{2000}{1}} = 63 \text{ m/s}$$

$$V_{\max}\sqrt{\frac{\rho}{W/S}} = 3.82 \qquad \therefore V_{\max} = 3.82\sqrt{\frac{2000}{1}} = 171 \text{ m/s}$$

The ordinate of point B, which is 1.25, establishes the smallest achievable turn radius $R_{\min}$ as follows:

$$\sqrt{\frac{R_{\min}g\rho}{W/S}} = 1.25 \qquad R_{\min} = \frac{(1.25)^2(2000)}{(9.81)(1)} = 319 \text{ m}$$

The abscissa value of 3.04 for point B gives

$$V = 3.04\sqrt{2000/1} = 136 \text{ m/s}$$

as the speed required to achieve this minimum turn radius.

To illustrate the use of Fig. 2, consider the same airplane, with the same limits on C_L and n, and establish the window of performance parameters permitting a turn rate ω of 0.24 rad/s (14 deg/s) or more. As before, points to the right of the $C_L = 1.3$ curve and to the left of the $n = 6$ line are permissible. The lower bound of 0.24 rad/s on the turn rate establishes the following upper bound on the ordinates in Fig. 2:

$$\frac{1}{\omega}\sqrt{\frac{\rho g^2}{W/S}} = \frac{1}{0.24}\sqrt{\frac{1(9.81)^2}{2000}} = 0.91$$

Thus, the performance window DEF is obtained. Points D and F, with abscissas of 1.88 and 5.38, respectively, define the minimum and maximum speeds as follows:

$$V_{\min} = 1.88\sqrt{2000/1} = 84\text{m/s}$$

$$V_{\max} = 5.38\sqrt{2000/1} = 241\text{m/s}$$

The ordinate 0.51 of point E establishes the maximum achievable turn rate $\omega_{\max}$ as follows:

$$\frac{1}{\omega_{\max}}\sqrt{\frac{\rho g^2}{W/S}} = 0.51 \qquad \therefore \omega_{\max} = \frac{1}{0.51}\sqrt{\frac{(1)(9.81)^2}{2000}} = 0.43 \text{ rad/s}$$

The abscissa of 3.0 for point E gives the speed required to achieve this turn rate as

$$V = 3.0\sqrt{2000/1} = 134\text{m/s}$$

The allowable range of load factors and lift coefficients, corresponding to either of the two velocity ranges determined above, is defined by the n and C_L associated with points A and C or D and F.

The use of dimensionless variables carries with it a benefit: It readily reveals how the turning performance is affected by a change in the wing loading or altitude. For example, from the ordinate label in Fig. 2, it is evident that, for specific C_L and n, the maneuverability as measured by ω varies inversely with $\sqrt{W/S}$ and directly with $\sqrt{\rho}$. Similarly, from Fig. 1, it is seen that maneuverability as measured by $1/R$ varies inversely with W/S and directly with ρ, assuming again that C_L and n are fixed.

The graphs also demonstrate the well-known fact that, for a given altitude and wing loading, maneuverability (large ω or small R) is improved by operating at the highest permissible lift coefficient and load factor. However, once the load factor reaches 3, it is clear from Fig. 1 that little additional reduction in R is achieved by any further increase in n. On the other hand, Fig. 2 shows that the turning rate ω is continually enhanced by increases in the permissible load factor.

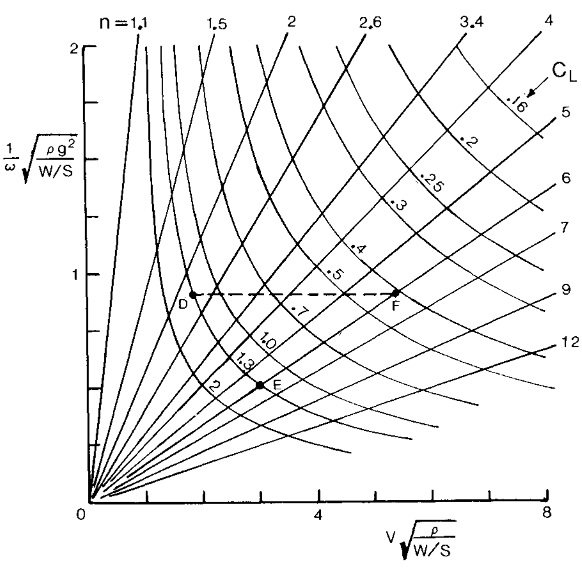

Fig. 2 ω-V-n-C_L relationships.

IV. Conclusion

The coordinated turn relationships have been presented graphically in such a way as to facilitate the delineation of "performance windows"—i.e., sets of operating conditions that will permit the attainment of specified turning performance, such as a specified maximum radius of turn or a specified minimum rate of turn. The dimensionless nature of the graphs also permits inferences to be drawn regarding the influence of altitude and wing loading on turning performance.

References

[1]Von Mises, R., *Theory of Flight*, McGraw-Hill Book Co., New York, 1945, p. 538.

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[3]Hale, F.J., *Introduction to Aircraft Performance, Selection, and Design*, John Wiley & Sons, New York, 1984.